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Polyakov loop correlators from D0-brane interactions in bosonic string theory*

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Abstract: In this paper we re-derive the effective Nambu-Goto theory result for the Polyakov loop correlator, starting from the free bosonic string and using a covariant quantization. The boundary conditions are those of an open string attached to two D0-branes at spatial distance R, in a target space with compact euclidean time. The one-loop free energy contains topologically distinct sectors corresponding to multiple covers of the cylinder in target space bordered by the Polyakov loops. The sector that winds once reproduces exactly the Nambu-Goto partition function. In our approach, the world-sheet duality between the open and closed channel is most evident and allows for an explicit interpretation of the free energy in terms of tree level exchange of closed strings between boundary states. Our treatment is fully consistent only in $d = 26$; extension to generic d may be justified for large R, and is supported by Montecarlo data. At shorter scales, consistency and Montecarlo data seem to suggest the necessity of taking into account the Liouville mode of Polyakov's formulation.

Keywords: Lattice Gauge Field Theories, Bosonic Strings, D-branes, Lattice QCD.

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Contents

1. Introduction

It is a long-standing belief [1] that the confining regime of non-abelian gauge theory should be described by an effective string theory describing the fluctuations of the color flux tube. Many theoretical insights and proposals have been put forward, while the development of lattice gauge theories (LGT) provides a better and better numerical test ground for the various models.

One of the main predictions which can be extracted from an effective string model and then tested in LGT simulations is the potential $V(R)$ between two external, massive quark and anti-quark sources in a pure glue theory. This potential can be obtained by considering, for instance, a rectangular Wilson loop $W(L, R)$ of sides L and R, for which $< W(L, R) >\sim \mathrm{e}^{-LV(R)}$ in the limit of large L. In the confining phase, the area law corresponds to a linear potential $V(R) = \mathcal{T} R + \ldots$. In a string interpretation, the area term $\mathcal{T} LR$ in the exponent of the Wilson loop is the classical action of the string model; $\mathcal{T}$ represents the string tension.[1] Upon quantization of the string model, we expect corrections to this classical potential.

In a seminal paper, Lüscher, Symanzik and Weisz [2], starting from the loop equations satisfied by the Wilson loops, derived the leading correction for large R. They found

$$V(R) = \mathcal{T}\, R - \frac{\pi}{24}\frac{d-2}{R} + O\left(\frac{1}{R^2}\right) . \tag{1.1}$$

Their computation, and subsequent ref. [3], linked this correction to the universal quantum contribution of $d-2$ massless modes corresponding to the transverse fluctuations of the string joining the quark–anti-quark pair. In this spirit, most of the theoretical calculations and of the comparisons with the lattice results in these last years were performed in

[1] We denote the string tension with $\mathcal{T}$ rather than with the usual notation σ to avoid confusion with the spatial coordinate of the string world sheet.

an *effective* description via the $c = d - 2$ two-dimensional conformal field theory of free bosons defined over the space-time surface of interest and with the appropriate boundary conditions; for instance, over the rectangle bordered by the Wilson loop, with Dirichlet boundary conditions. The picture may be refined by allowing a set of higher order interactions among the fields of the theory, whose precise form distinguishes the various effective theories. In a string picture, the underlying string model should determine a specific form of the effective theory, and an expression of the potential that extends eq. (1.1) to finite values of R.

An observable which presents a lot of interest in this respect is the correlator of two Polyakov loops at spatial distance R in the gauge theory at finite temperature $1/L$. On the one hand, this quantity can be measured with very high precision on the lattice. On the other hand, in a string interpretation, the correlation is due to the cylindric world-sheet spanned by the stretched string and is therefore associated to the partition function of the effective string model, and not just to its ground state energy $V(R)$. The Polyakov loop is thus very useful to discriminate between different models.

The simplest and most natural string model is the Nambu-Goto [4] one. For the Nambu-Goto string with boundary conditions corresponding to fixed ends in the spatial directions (the static quark and anti-quark) Alvarez [5] (for $d \to \infty$) and Arvis [6], with a formal quantization, obtained the energy spectrum

$$E_n(R) = \mathcal{T}R\sqrt{1 + \frac{2\pi}{\mathcal{T}R^2}\left(n - \frac{d-2}{24}\right)}\,. \tag{1.2}$$

The static potential equals the lowest energy level: $V(R) = E_0(R)$, reproducing eq. (1.1) for large R. The partition function is thus

$$Z = \sum_n w_n e^{-LE_n(R)}\,, \tag{1.3}$$

w_n being the usual multiplicities of the bosonic string. The derivation of eqs. (1.2) and (1.3) in [6] uses the re-parametrization invariance of the world-sheet to reach the conformal gauge (where the Nambu-Goto action is equivalent to the free string action) and the residual conformal invariance to fix a light-cone type gauge (which is sometimes denoted as "physical gauge"). This leaves as only independent dynamical variables the transverse modes, which become oscillators upon quantization. The energy, in particular, is re-expressed in terms of the occupation number n of these oscillators and the spectrum eq. (1.2), as remarked in [7], stems from the analogue of the standard mass formula $M^2 = \mathcal{T}[n - (d-2)/24]$ for the bosonic open string with free ends.

This bosonic string model, of course, is truly consistent at the quantum level only if $d = 26$: as usual in light-cone type gauges, Lorentz invariance is otherwise broken. It was however noticed in [7] that the coefficient of the anomaly vanishes for $R \to \infty$, so that in this regime the model could be consistent.

In these last years, thanks to various remarkable improvements in lattice simulations [8 − 11] the effective string picture could be tested with a very high degree of precision and confidence [11]–[27]. The picture which emerges is that at large inter-quark distances

JHEP07(2005)038

and low temperatures the Nambu-Goto effective string eq. (1.2) correctly describes the Montecarlo data. Moreover, this result is universal, meaning that it does not depend on the particular gauge group under study (the same behaviour is observed in models as different as the $\mathbb{Z}_2$ gauge model in $(2+1)$ dimensions [16] and the SU(3) LGT in $(3+1)$ dimensions [12]). As the inter-quark distance decreases and/or the temperature increases (i.e. as the de-confinement transition is approached) clear deviations from this picture are observed and the universality mentioned above is partially lost [14, 16−18].

These deviations could well be connected to the inconsistency of the model at $d < 26$ becoming more and more relevant as R decreases. A consistent quantum formulation in $d < 26$ can be sought in the Polyakov formulation [28]. We will briefly comment on this possibility in sec. (3).

In this paper, we re-derive the effective Nambu-Goto theory, and in particular the result eq. (1.3) for the Polyakov loop correlator, starting from the free bosonic string and using a covariant quantization.[2] The boundary conditions (the same as in Arvis' paper) are described in modern terms as those of an open string attached to two D0-branes at spatial distance R. We work at finite temperature, i.e., in a target space with compact euclidean time, and compute the free energy for such open strings. This is nothing but the well-known Polchinski derivation of the interaction between two D-branes [31], adapted to the present case. We do not impose any light-cone-like or physical gauge, so we keep the string world-sheet distinct from the target-space surface bordered by the Polyakov loops. In fact, the free energy contains different topological sectors corresponding to multiple covers of the cylinder in target space bordered by the Polyakov loops. The sector that winds once reproduces exactly eq. (1.3). Thus, the expression of the Polyakov loop correlator and the inter-quark potential obtained from the covariant quantization of the free bosonic string is the one of the Nambu-Goto effective string, and not the one[3] obtained from the effective free bosonic string, i.e. the CFT of $d-2$ free bosons which is the simplest element in the universality class of eq. (1.1).

Just as for Arvis, our treatment is fully consistent only in $d = 26$; extension to generic d may be justified for large R, and is supported by Montecarlo data as already described above.

We think that the re-derivation of the Nambu-Goto effective theory presented here may have some advantages. It dwells on a standard computation in the framework of the free bosonic string, the open string free energy at one loop, for which simple operatorial methods are effective. Not having fixed a "physical gauge", the world-sheet duality between the open and closed channel is most evident and allows for an explicit interpretation of

[2] In a covariant quantization, the conformal invariance of the free string model is not gauge-fixed by identifying the world-sheet coordinates with some directions in the target space. The constraints corresponding to the conformal symmetry (Virasoro constraints) are imposed at the quantum level *á la* Gupta-Bleuer (old covariant quantization) or via BRST quantization. See, for instance, [29] or [30] for a review; some more details are recalled here in section 2.

[3] In this case, the energy spectrum corresponds to the second-order truncation of the square-root in eq. (1.2), yielding a partition function which is simply $Z_{(0)} = \mathrm{e}^{\mathcal{T}LR}\eta(\mathrm{i}\,\frac{L}{2R})$, η being the well known Dedekind function.

the free energy in terms of tree level exchange of closed string states between boundary states. Our formulation is well suited in principle to study the contributions to the inter-quark potential from string interactions, which in our language would mean wrapping the Polyakov string on surfaces (bordered by the Polyakov loops) with handles, as well as to investigate different observables such as Wilson loops or interfaces. It could also be interesting to investigate the possible relevance in the gauge theory of the contributions to the free energy with different winding numbers. Finally, since the covariant treatment basically coincides with Polyakov's formulation upon neglecting the Liouville field, one may try to correct its results at finite scale R by including the effect of the Liouville theory, see the brief discussion in section 3.

This paper is organized as follows. Section 2 contains the main computation. In particular, in subsection 2.1 we compute the open string free energy, in subsection 2.2 we perform its modular transformation and in subsection 2.3 we give a detailed re-interpretation of the modular transformed expression in the closed string channel. In section 3 some conclusions and speculations are presented.

2. Polyakov loop correlators and strings between D0-branes

The Polyakov loop is the trace of the temporal Wilson line induced by the presence of a static quark minimally coupled to the non-abelian gauge field. In a stringy perspective, the quark represents the end-point of a string. The v.e.v. of $P(\vec{R})$ is related to the free energy of this static quark: $\langle P(\vec{R})\rangle = \mathrm{e}^{-F}$, and a non-zero value for this v.e.v. signals de-confinement, as having an isolated quark requires a finite energy.

The Polyakov loop is the order parameter of the $\mathbb{Z}_N$ global symmetry which appears when the $\mathrm{SU}(N)$ gauge theory is regularized on a lattice with a finite-temperature geometry (i.e. periodic boundary conditions in the "time" direction). This additional symmetry coincides with the center of the original gauge group and is broken in the de-confined phase.

An observable which can be measured with great accuracy on the lattice is the connected correlator of two spatially separated Polyakov loops,

$$\langle P(\vec{0})P(\vec{R})\rangle_{\mathrm{c}}\,. \tag{2.1}$$

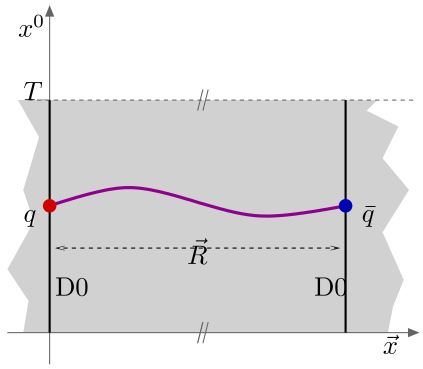

Figure 1: Let us consider two Polyakov loops at spatial distance $\vec{R}$. An open string connecting the two external static quarks obeys boundary conditions corresponding to D0-branes on a compact space.

In the string picture, see figure 1, the correlation is due to the strings connecting the two external, static quark and anti-quark that span the two Polyakov loops. We take the point of view that such strings can be described by the standard bosonic string theory in the d-dimensional space-time under consideration, which is flat, but with compact Euclidean time

$$x_0 \sim x_0 + L\,. \tag{2.2}$$

Using the first order formulation, the string action in the conformal gauge reads simply

$$S = \frac{1}{4\pi\alpha'} \int d\tau \int_0^\pi d\sigma \, \left[(\partial_\tau X^M)^2 + (\partial_\sigma X^M)^2 \right] + S_{\text{gh.}} \,, \tag{2.3}$$

where $\sigma \in [0, \pi]$ parametrizes the spatial extension of the string and τ its proper time evolution. The string tension $\mathcal{T}$ is given in this notation by

$$\mathcal{T} = \frac{1}{2\pi\alpha'} \,. \tag{2.4}$$

The fields $X^M(\tau, \sigma)$, with $M = 0, \ldots, d-1$, describe the embedding of the string world-sheet in the target space and form the 2-dimensional CFT of d free bosons. The term $S_{\text{gh.}}$ in eq. (2.3) is the action for the ghost and anti-ghost fields (traditionally called c and b) that arise from the jacobian for fixing the conformal gauge. We do not really need here its explicit expression, see [29] or [30] for reviews. In the conformal gauge the world-sheet metric is of the form $g_{\alpha\beta} = \mathrm{e}^{\phi} \delta_{\alpha\beta}$, and corresponds to a CFT of central charge $c_{\text{gh.}} = -26$. The scale factor e^{ϕ} decouples at the classical level, but this property persists at the quantum level only if the anomaly parametrized by the total central charge $c = d - 26$ vanishes. We will nevertheless proceed in the case of general d, according to the discussion in the introduction.

The open string joining the q-$\bar{q}$ static quarks as in figure 1 obeys Neumann boundary conditions at both ends in the time direction:

$$\partial_\sigma X^0(\tau, \sigma)\big|_{\sigma=0,\pi} = 0 \,, \tag{2.5}$$

while it satisfies Dirichlet boundary conditions in the spatial directions:

$$\vec{X}(\tau, 0) = 0 \,, \qquad \vec{X}(\tau, \pi) = \vec{r} \,. \tag{2.6}$$

These conditions constrain the endpoints to two lines (1-dimensional "branes") which are nowadays known as D0-branes. The mode expansion of the X^M fields with such boundary conditions is

$$\begin{aligned} X^0(\tau, \sigma) &= \hat{x}^0 + 2\alpha' \hat{p}^0 \tau + \mathrm{i}\sqrt{2\alpha'} \sum_{n \neq 0} \frac{\alpha_n^0}{n} \mathrm{e}^{-\mathrm{i}n\tau} \cos n\sigma \,, \\ \vec{X}(\tau, \sigma) &= \frac{\vec{R}}{\pi} \sigma - \sqrt{2\alpha'} \sum_{n \neq 0} \frac{\vec{\alpha}}{n} \mathrm{e}^{-\mathrm{i}n\tau} \sin n\sigma \,. \end{aligned} \tag{2.7}$$

Upon canonical quantization, the oscillators α_n^M obey the algebra

$$\left[\alpha_m^M, \alpha_n^N \right] = m \, \delta_{m+n,0} \, \delta^{MN} \,. \tag{2.8}$$

The eigenvalues p^0 of the momentum operator $\hat{p}^0$ are discrete, because of the periodicity eq. (2.2):

$$\hat{p}^0 \to \frac{2\pi n}{L} \,. \tag{2.9}$$

The generators of the conformal transformations are called Virasoro generators and traditionally indicated as L_m. In particular, L_0 generates the world-sheet dilations and corresponds to the hamiltonian derived from the action eq. (2.3). It receives therefore contributions from the bosons and the ghost system: $L_0 = L_0^{(X)} + L_0^{(\mathrm{gh.})}$, and we have

$$L_0^{(X)} = \alpha' (\hat{p}^0)^2 + \frac{R^2}{4\pi^2 \alpha'} + \sum_{n=1}^{\infty} N_n - \frac{d}{24}\,, \tag{2.10}$$

where $N_n = \sum_M \alpha_{-n}^M \cdot \alpha_n^M$ is the occupation number for the oscillators α_n^M, and $d/24$ is the (ζ function regularized[4]) normal ordering constant. The contribution $R^2/(4\pi^2\alpha')$ represents the energy needed to stretch the string between the two branes. For the b, c ghost system we have, see for instance [29],

$$L_0^{\mathrm{gh}} = \text{non-zero modes} + \frac{1}{12}\,. \tag{2.11}$$

The b, c, are indeed anti-commuting bosonic fields, and they get a normal ordering constant of the opposite sign with respect to the X's and, in the trace of eq. (2.12) they will contribute exactly the inverse of the non-zero mode part of two bosonic directions.

2.1 Open string free energy

Let us now compute the one-loop free energy of the (non-critical) open strings with their endpoints attached to the two different D0-branes. The expression to be considered is[5]

$$\mathcal{F} = L \int_0^{\infty} \frac{dt}{2t}\, \mathrm{Tr}\, q^{L_0}\,. \tag{2.12}$$

In this expression, we integrate over the single real modulus t of the world-sheet surface, which is a cylinder, as we have to do in our first-order formulation. The factor L represents the volume of the only target space direction along which the excitations can propagate, namely the Euclidean time. In eq. (2.12) we introduced

$$q = \exp(-2\pi t)\,. \tag{2.13}$$

The trace in eq. (2.12) decomposes in a trace over non-zero modes and a zero-mode part. For the non-zero modes, including also the normal-ordering constant, we get for each bosonic direction the usual result

$$q^{-\frac{1}{24}} \prod_{r=1}^{\infty} \frac{1}{1-q^r} = \frac{1}{\eta(\mathrm{i}t)}\,. \tag{2.14}$$

The only operatorial zero mode is the momentum $\hat{p}^0$ appearing in the X^0 field; the distance $\vec{r}$ between the branes appears instead as a numerical parameter in the expansion of the $\vec{X}$ fields. Since the 0-th direction is compactified according to eq. (2.2), the

[4] In ref. [32] some interesting words of caution were raised regarding the use of ζ-function regularization for the string with the present boundary conditions.

[5] We consider a given orientation of the string.

eigenvalues of p^0 are quantized, see eq. (2.9): $p^0 = 2\pi n/L$. The corresponding trace, which in the non-compact case is given by an integral, requires therefore the discrete sum $(1/L)\sum_n$.

Taking into account also the zero-mode contributions to L_0 and the relation, eq. (2.13), between q and t the free energy eq. (2.12) is expressed as

$$\mathcal{F} = \int_0^\infty \frac{dt}{2t} \sum_{n=-\infty}^{\infty} \mathrm{e}^{-2\pi t\left(\frac{4\pi^2\alpha' n^2}{L^2}+\frac{R^2}{4\pi^2\alpha'}\right)} \left(\frac{1}{\eta(\mathrm{i}t)}\right)^{d-2}, \tag{2.15}$$

where the exponent of $d-2$ for the non-zero mode trace is due to the fact that the ghost contribution cancels exactly the non-zero modes of two bosonic directions. In the effective interpretation, these two coordinates are the time one, X^0, and one of the spatial ones; as a result, $d-2$ spatial transverse coordinates are left.

We can now Poisson re-sum over the integer n labelling the momentum:

$$\sum_{n=-\infty}^{\infty} \exp\left(-\frac{8\pi^3\alpha' t}{L^2} n^2\right) = \sqrt{\frac{L^2}{8\pi^2\alpha' t}} \sum_{m=-\infty}^{\infty} \exp\left(-\frac{L^2}{8\pi\alpha' t} m^2\right). \tag{2.16}$$

In this dual expansion, the integer m labels the topologically distinct sectors in which the string world-sheet winds m times around the compact time direction. Notice that winding in one direction or the opposite yields the same contribution, as only m^2 occurs. We can thus write

$$\mathcal{F} = \mathcal{F}^{(0)} + 2\sum_{m=1}^{\infty} \mathcal{F}^{(m)}, \tag{2.17}$$

with

$$\mathcal{F}^{(m)} = \frac{L}{\sqrt{8\pi^2\alpha'}} \int_0^\infty \frac{dt}{2t^{\frac{3}{2}}} \mathrm{e}^{-\frac{L^2 m^2}{8\pi\alpha' t} - t\frac{R^2}{2\pi\alpha'}} \left(\frac{1}{\eta(\mathrm{i}t)}\right)^{d-2}. \tag{2.18}$$

In the case $m = \pm 1$, the string end-points trace out in the target space the two Polyakov loops, and the string world-sheet has exactly the topology of the cylinder bordered by the Polyakov loop whose fluctuations are assumed to be described by the “effective” Nambu-Goto theory in the usual treatment. Let us see how our computation, in this topological subsector, is related to such a description.

First of all, in eq. (2.18) we expand in series of q the infinite products in eqs. (2.14), (2.15):

$$\left(\prod_{r=1}^{\infty} \frac{1}{1-q^r}\right)^{d-2} = \sum_{k=0}^{\infty} w_k q^k \tag{2.19}$$

(for $d = 3$ we have simply $w_k = p_k$, the number of partitions of the integer k). Having done this, the integration over t in eq. (2.18) can be performed[6] obtaining, in the case $m \neq 0$,

$$\mathcal{F}^{(m)} = \frac{1}{2|m|} \sum_k w_k \mathrm{e}^{-|m| L E_k(R)}, \qquad (m \neq 0), \tag{2.21}$$

[6]In the case $m \neq 0$ we use the integral

$$\int_0^\infty \frac{dt}{t^{\frac{3}{2}}} \mathrm{e}^{-\frac{\alpha^2}{t} - \beta^2 t} = \frac{\sqrt{\pi}}{|\alpha|} \mathrm{e}^{-2|\alpha|\,|\beta|} \tag{2.20}$$

with $\alpha^2 = L^2 m^2/(8\pi\alpha')$ and $\beta^2 = \frac{R^2}{2\pi\alpha'} + 2\pi(k - \frac{d-2}{24})$.

where

$$E_k(r) = \frac{R}{4\pi\alpha'}\sqrt{1+\frac{4\pi^2\alpha'}{R^2}\left(k-\frac{d-2}{24}\right)} \tag{2.22}$$

are nothing else but the Nambu-Goto energy levels of Alvarez and Arvis, see eq. (1.2). In particular, from the $m = \pm 1$ cases we get

$$2\mathcal{F}^{(1)} = Z(R)\,, \tag{2.23}$$

where $Z(R)$ is the Nambu-Goto partition function of eq. (1.3).

The case $m = 0$ corresponds exactly (apart from the volume factor L being finite) to the usual result one gets in the non-compact situation:

$$\mathcal{F}^{(0)} = L\int_0^\infty \frac{dt}{2t}\frac{1}{\sqrt{8\pi^2\alpha' t}}\left(\frac{1}{\eta(\mathrm{i}t)}\right)^{d-2} \mathrm{e}^{-\frac{tR}{2\pi\alpha'}}\,, \tag{2.24}$$

see for instance [33]. Using the expansion eq. (2.19), the integration over t can be easily carried out, with the result

$$\mathcal{F}^{(0)} = -L\sum_k w_k E_k(R)\,. \tag{2.25}$$

2.2 Modular transformation to the closed channel

Changing integration variable to $s = 1/t$ and taking advantage of the modular properties of the Dedekind eta function:

$$\eta(\mathrm{i}/s) = s^{\frac{1}{2}}\,\eta(\mathrm{i}s) \tag{2.26}$$

the expression eq. (2.18) of the free energy in the m-th sector can be written as

$$\begin{aligned}\mathcal{F}^{(m)} &= \frac{L}{\sqrt{8\pi^2\alpha'}}\int_0^\infty \frac{ds}{2s} s^{\frac{3-d}{2}}\mathrm{e}^{-\frac{L^2m^2}{8\pi\alpha'}s-\frac{R^2}{2\pi\alpha' s}}\left(\frac{1}{\eta(\mathrm{i}s)}\right)^{d-2} \\ &= \frac{L}{\sqrt{8\pi^2\alpha'}}\sum_k w_k\int_0^\infty \frac{ds}{2s}s^{\frac{3-d}{2}}\mathrm{e}^{-\left[\frac{L^2m^2}{8\pi\alpha'}+2\pi\left(k-\frac{d-2}{24}\right)\right]s-\frac{R^2}{2\pi\alpha' s}}\,. \end{aligned} \tag{2.27}$$

In terms of the variable $z = \pi\alpha' s/2$ this becomes

$$\mathcal{F}^{(m)} = \frac{L}{\sqrt{8\pi^2\alpha'}}\left(\frac{2}{\pi\alpha'}\right)^{\frac{3-d}{2}}\sum_k w_k\int_0^\infty \frac{dz}{2z}z^{\frac{3-d}{2}}\mathrm{e}^{-M^2(m,k)\,z-\frac{R^2}{4z}}\,, \tag{2.28}$$

with

$$M^2(m,k) = \frac{4}{\alpha'}\left(k-\frac{d-2}{24}\right)+\left(\frac{mL}{2\pi\alpha'}\right)^2 \tag{2.29}$$

$$= (m\mathcal{T}L)^2\left[1+\frac{8\pi}{\mathcal{T}L^2m^2}\left(k-\frac{d-2}{24}\right)\right]. \tag{2.30}$$

In the second line above we wrote the expression in such a way that it can be easily compared, for $m = 1$, with [13, eq. (C5)], see the discussion at the end of this section.

The integral appearing in eq. (2.28) is proportional to the propagator of a scalar field of mass M^2 over the distance $\vec{R}$ between the two D0-branes along the $d-1$ spatial directions. Indeed, such a propagator is given by

$$\begin{aligned} G(R;M) &= \int \frac{d^{d-1}p}{(2\pi)^{d-1}} \frac{\mathrm{e}^{\mathrm{i}\vec{p}\cdot\vec{R}}}{p^2+M^2} = \int_0^\infty dz \int \frac{d^{d-1}p}{(2\pi)^{d-1}} \mathrm{e}^{-z(p^2+M^2)+\mathrm{i}\vec{p}\cdot\vec{r}} \\ &= \frac{1}{(4\pi)^{\frac{d-1}{2}}} \int_0^\infty \frac{dz}{z} z^{\frac{3-d}{2}} \mathrm{e}^{-M^2 z - \frac{R^2}{4z}} = \frac{1}{2\pi}\left(\frac{M}{2\pi R}\right)^{\frac{d-3}{2}} K_{\frac{d-3}{2}}(MR)\,. \end{aligned} \tag{2.31}$$

The free energy eq. (2.28) can therefore be seen as a collection of tree-level exchange diagrams between the D0-branes; the exchanged particles have squared mass $M^2(k,m)$ given by eq. (2.30).

This picture nicely agrees with what dimensional reduction and the so called Svetitsky-Yaffe conjecture [34] suggest on the behaviour of the Polyakov loop correlator as the de-confinement temperature is approached from below. According to this conjecture, in the vicinity of the de-confinement point, if the transition is continuous, a d-dimensional LGT with gauge group G can be effectively described by a $d-1$ dimensional spin model with symmetry group the center of G. In this representation the Polyakov loops become the spins of the underlying model and the Polyakov loop correlator is nothing else than the standard spin-spin correlator.[7] This correlator will depend on the symmetry group and will be in general very complicated; however, at large distance it will be dominated by the contribution of the lowest mass particle in the spectrum, whose contribution to the correlation function, in $d-1$ dimensions, will be represented exactly by the Bessel function $K_{\frac{d-3}{2}}(mR)$, with $m \equiv M(1,0)$ being the mass of this particle. Thus the Nambu-Goto string description is fully compatible with our qualitative understanding of the high temperature behaviour of the gauge theory.

Collecting all the numerical factors, we can write

$$\mathcal{F}^{(m)} = L\frac{T_0^2}{4}\sum_k w_k\, G\left(R; M(m,k)\right). \tag{2.32}$$

where T_0 is the 0-brane tension, in accordance with the standard result for the tension T_p of Dp-branes in bosonic string theory, see for instance [33, 36]:

$$T_p = \sqrt{\frac{\pi}{2^{\frac{d}{2}-5}}}\left(2\pi\sqrt{\alpha'}\right)^{\frac{d}{2}-2-p}, \tag{2.33}$$

which, for $p=0$, gives

$$T_0^2 = 8\pi\left(\frac{\pi}{\mathcal{T}}\right)^{\frac{d}{2}-2}. \tag{2.34}$$

The exchanged states are *closed string states*, with k representing the total oscillator number, and m the wrapping number of the string around the compact time direction.

[7] Notice as a side remark that the closed string channel discussed here is the one which better describes the high temperature behaviour of the Polyakov loop correlators where L is in general much smaller than the interquark distance R.

JHEP07(2005)038

The modular transformation to the closed channel of the case $m = 1$, i.e., for the NG partition function eq. (1.3), was performed in [13]; let us compare our findings to that reference. The mass M of the fields exchanged by the D_0 branes, see eq. (2.30), coincide for $m = 1$ with the "closed string energies" reported in [13, eq. (C5)]. With this identification eqs. (2.31) and (2.32) exactly coincide with [13, eq. (3.2)]. This allows to relate the "transition matrix elements" reported in [13, eq. (C.6)] with the tension T_0 of the two $D0$ branes.

It is clear from the above identifications that our derivation is fully equivalent to the original one by Lüscher and Weisz [13]. The only advantage of our formulation is that it allows us to describe the result of this mathematical transformation directly in the closed string formulation, as we will see in the next section, sheding some more light on the string interpretation of the transformation.

2.3 Closed string interpretation

Let us now re-derive the Nambu-Goto partition function in the closed string channel. This requires the introduction of the notion of "boundary state", which plays a major role in the following, and could be relevant for the stringy treatment of other gauge geometries.[8]

The modular transformation of the cylinder amplitude corresponds to the interchange of the world-sheet coordinates $\sigma \leftrightarrow \tau$, so that the loop of an open string gets re-interpreted as the tree level propagation of a closed string between two *boundary* states representing the two D0-branes:

$$\mathcal{F} = \langle B; \vec{0}\,|\mathcal{D}|B; \vec{R}\,\rangle\,, \tag{2.35}$$

where $\mathcal{D}$ is the closed string propagator, see below. An explicit expression of the boundary states makes it possible to derive the closed-channel expression eq. (2.27) of the free energy $\mathcal{F}$ entirely within the closed string formalism.

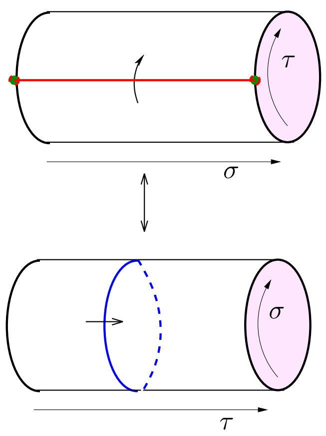

Figure 2: The interchange $\sigma \leftrightarrow \tau$ maps the open string 1-loop free energy (upper) to tree level propagation of a closed string (lower).

The closed string fields $X^i(\tau, \sigma)$ in the spatial directions have the standard mode expansion

$$X^i(\tau, \sigma) = \hat{x}^i + \alpha'\,\hat{p}^i\,\tau + \mathrm{i}\sqrt{\frac{\alpha'}{2}} \sum_{n\neq 0} \left(\frac{\alpha_n^i}{n} \mathrm{e}^{-\mathrm{i}n(\tau+\sigma)} + \frac{\widetilde{\alpha}_n^i}{n} \mathrm{e}^{-\mathrm{i}n(\tau-\sigma)} \right). \tag{2.36}$$

In the compact time direction, which has the periodicity eq. (2.2), the zero-mode sector is modified:

$$X^0(\tau, \sigma) = \hat{x}^0 + \alpha'\,\hat{p}^0\,\tau + \frac{\hat{m}L}{2\pi}\,\sigma + \text{non-zero modes}\,, \tag{2.37}$$

where the momentum operator $\hat{p}^0$ has quantized eigenvalues $2\pi n/L$ and $\hat{m}$ has integer eigenvalues m corresponding to the *winding number* of the string in the time direction:

$$X^0(\tau, \sigma + 2\pi) = X^0(\tau, \sigma) + mL\,. \tag{2.38}$$

[8] See [35] for a list of references in the context of string and superstring theory; ref. [36] is a very useful review. See [37] for a discussion of boundary states on compact target spaces, which is relevant here for the X^0 direction. In ref. [38] we give some basic references regarding boundary states in the context of CFT's.

The left- and right-moving oscillators α_n^M and $\widetilde{\alpha}_n^M$, with $M = (0, i)$, satisfy the algebra

$$\left[\alpha_m^M, \alpha_n^N\right] = m\,\delta_{m+n,0}\delta^{MN} \tag{2.39}$$

(and the analogous one for the right-movers). The left-moving and right-moving Virasoro generators L_0 and $\widetilde{L}_0$ are given by

$$L_0 = \frac{\alpha'}{4}\sum_i(\hat{p}^i)^2 + \frac{\alpha'}{4}\left(\hat{p}^0 + \frac{\hat{m}L}{2\pi\alpha'}\right)^2 + \sum_{n=1}^{\infty}\alpha_{-n}\cdot\alpha_n - \frac{d}{24}\,, \tag{2.40}$$

$$\widetilde{L}_0 = \frac{\alpha'}{4}\sum_i(\hat{p}^i)^2 + \frac{\alpha'}{4}\left(\hat{p}^0 - \frac{\hat{m}L}{2\pi\alpha'}\right)^2 + \sum_{n=1}^{\infty}\widetilde{\alpha}_{-n}\cdot\widetilde{\alpha}_n - \frac{d}{24}\,, \tag{2.41}$$

where we included the normal ordering constants. A standard form of the closed string propagator is given by

$$\frac{\alpha'}{4\pi}\int\frac{d^2z}{|z|^2}z^{L_0+L_0^{\text{gh.}}}\,\bar{z}^{\widetilde{L}_0+\widetilde{L}_0^{\text{gh.}}}\,, \tag{2.42}$$

where $L_0^{\text{gh.}}$ and $\widetilde{L}_0^{\text{gh.}}$ are the Virasoro generators for the ghost system.

The *boundary state* $|B;\vec{R}\rangle$ appearing in eq. (2.35) represents in the closed string language a D0-brane located in its transverse directions at $\vec{R}$. It is a state in the Hilbert space of the closed string which inserts a boundary in the world-sheet at $\tau = 0$ and enforces the closed string counterparts of the open string boundary conditions eqs. (2.5), (2.6), Neumann along the time, Dirichlet along the spatial directions:

$$\partial_\tau X^0(\sigma,\tau)\big|_{\tau=0}|B;\vec{R}\rangle = 0\,, \qquad \left(X^i(\sigma,\tau) - R^i\right)\big|_{\tau=0}|B;\vec{R}\rangle = 0\,. \tag{2.43}$$

In terms of the mode expansions eq. (2.36) and eq. (2.37), these conditions become

$$\left(\alpha_n^0 + \widetilde{\alpha}_{-n}^0\right)|B;\vec{R}\,\rangle = 0\,, \qquad \hat{n}|B;\vec{R}\,\rangle = 0 \tag{2.44}$$

in the time directions, and

$$\left(\alpha_n^i - \widetilde{\alpha}_{-n}^i\right)|B;\vec{R}\,\rangle = 0\,, \qquad \left(\hat{x}^i - r^i\right)|B;\vec{R}\,\rangle = 0\,. \tag{2.45}$$

in the spatial ones. It follows from these conditions that the boundary state also identifies the left- and right-moving Virasoro generators:

$$\left(L_0 - \widetilde{L}_0\right)|B;\vec{R}\,\rangle = 0\,. \tag{2.46}$$

Requiring the BRST invariance of the boundary state implies that it also has a component in the ghost sector; for the details we refer, for instance, to [36]. Let us just notice that the boundary state identifies also the left- and right-moving ghost Virasoro generator $L_0^{\text{gh}}, \widetilde{L}_0^{\text{gh}}$, analogously to eq. (2.46).

The solution to these conditions has the form

$$|B;\vec{R}\,\rangle = \mathcal{N}_0\,|B^{(0)};\vec{R}\,\rangle|B^{\text{n.z.}}\rangle|B^{\text{gh.}}\rangle\,, \tag{2.47}$$

where $\mathcal{N}_0$ is a normalization to be fixed. The non-zero-mode part of the boundary state reads explicitly

$$|B^{\mathrm{n.z.}}\rangle = \exp\left\{-\sum_{n=1}^{\infty}\frac{1}{n}\left(\alpha^0_{-n}\widetilde{\alpha}^0_{-n} - \sum_i \alpha^i_{-n}\cdot\widetilde{\alpha}^i_{-n}\right)\right\}|0,\widetilde{0}\rangle\,. \tag{2.48}$$

Since the Neumann conditions eq. (2.44) leave the winding m undetermined, the zero-mode part of the boundary state decomposes as $|B^{(0)};\vec{R}\,\rangle = \sum_m |B^{(0)};\vec{R}\,;m\rangle$, the m-th component belonging to the sub-sector of the Hilbert space that describes strings wrapped m times around the time circle. We have

$$|B^{(0)};\vec{R}\,;m\rangle = \delta(\hat{\vec{x}}-\vec{R})|n=0,m;\vec{p}=0\rangle = \int\frac{d^{d-1}p}{(2\pi)^{d-1}}\mathrm{e}^{-\mathrm{i}\vec{p}\cdot\vec{R}}|n=0,m;\vec{p}\rangle\,, \tag{2.49}$$

where in the second step we made explicit the δ-function along the Dirichlet directions and introduced momentum eigenstates[9] $|\vec{p}\rangle$.

Taking into account the identification of the left- and right-moving Virasoro generators on the boundary states, the amplitude eq. (2.35) becomes, introducing $|z| = \exp(-\pi s)$,

$$\mathcal{F} = \frac{\pi\alpha'}{2}\int_0^\infty ds\,\langle B;\vec{0}\,|\mathrm{e}^{-2\pi s(L_0+L_0^{\mathrm{gh.}})}|B;\vec{R}\,\rangle\,. \tag{2.50}$$

The matrix element for the non-zero mode sector of the above expression can be easily computed using the oscillator algebra eq. (2.39) and the properties of coherent-like states such as the ones appearing in eq. (2.48), see [36]. The result is

$$\langle B^{\mathrm{n.z.}}|\mathrm{e}^{-2\pi s\left(\sum_{k=1}^{\infty}\alpha_{-k}\cdot\alpha_k-\frac{d}{24}\right)}|B^{\mathrm{n.z.}}\rangle = \mathrm{e}^{\frac{\pi d s}{12}}\prod_{n=1}^{\infty}\left(\frac{1}{1-\mathrm{e}^{-2\pi n s}}\right)^d = \left(\frac{1}{\eta(\mathrm{i}s)}\right)^d. \tag{2.51}$$

The matrix element in the ghost sector effectively cancels out the contributions of the bosonic non-zero modes of two directions:

$$\langle B^{\mathrm{gh.}}|\mathrm{e}^{-2\pi s L_0^{\mathrm{gh.}}}|B^{\mathrm{gh.}}\rangle = \eta^2(\mathrm{i}s)\,. \tag{2.52}$$

The matrix element in the zero-mode sector, using eq. (2.49) and the 0-mode part of L_0 as given in eq. (2.40), becomes, for each winding m,

$$\begin{aligned}\int\frac{d^{d-1}q}{(2\pi)^{d-1}}\int\frac{d^{d-1}p}{(2\pi)^{d-1}}\langle n=0,m;\vec{q}|\mathrm{e}^{-\frac{\pi\alpha' s}{2}\left(\sum_i(\hat{p}^i)^2+\left(\hat{p}^0+\frac{\hat{m}L}{2\pi\alpha'}\right)^2\right)-\mathrm{i}\vec{p}\cdot\vec{R}}|n=0,m;\vec{p}\rangle =\\ = (2\pi^2\alpha')^{-\frac{d-1}{2}}\mathrm{e}^{-\frac{m^2L^2s}{8\pi\alpha'}-\frac{r^2}{2\pi\alpha' s}}\,,\end{aligned} \tag{2.53}$$

where in the last step we used the orthogonality of the momentum and winding eigenstates, and performed the remaining gaussian integration.

[9] These states we normalize to $\langle\vec{k}|\vec{p}\rangle = (2\pi)^{d-1}\delta^{d-1}(\vec{k}-\vec{p})$. The quantized momentum and winding $|n\rangle$ and $|m\rangle$ in the time direction are instead simply normalized to the Kronecker δ.

JHEP07(2005)038

The amplitude $\mathcal{F} = \sum_m \mathcal{F}^{(m)}$ receives contributions from all winding sectors and, collecting all the ingredients, we find

$$\mathcal{F}^{(m)} = \mathcal{N}_0^2 \left(2\pi^2\alpha'\right)^{\frac{1-d}{2}} \frac{\pi\alpha'}{2} \int_0^\infty \frac{ds}{s} s^{\frac{3-d}{2}} \mathrm{e}^{-\frac{m^2L^2 s}{8\pi\alpha'} - \frac{R^2}{2\pi\alpha' s}} \left(\frac{1}{\eta(\mathrm{i}s)}\right)^{d-2} . \tag{2.54}$$

The boundary state normalization has to be chosen so as to agree with the modular transformation, eq. (2.27), of the open channel cylinder trace. This fixes

$$\mathcal{N}_0 = \frac{T_0}{2}\sqrt{L}\,, \tag{2.55}$$

where the tension T_p of the bosonic Dp-brane in the non-compact theory was already given in eq. (2.33).

3. Conclusions

In this paper we have shown how to derive the Nambu-Goto effective string from a covariant quantization of the bosonic string in d dimensions, which is tantamount to the Polyakov formulation if one neglects the Liouville field.

As we noticed in the introduction, for large enough values of R and L the Nambu-Goto string is in very good agreement with the results of Montecarlo simulations for various gauge theories and in various dimensions, see for instance figure 3 and 4 (taken with small changes from [16]) referring to the $\mathbb{Z}_2$ gauge theory in (2+1) dimensions. Work is in progress [39] to quantify this agreement also at the level of the string spectrum, and the preliminary results are very favourable to the Nambu-Goto effective model. This suggests that indeed at large distance one can neglect the Liouville mode and still correctly describe the fluctuations of the flux tube which joins together the quark–anti-quark pair by standard bosonic string theory. As it has been observed by several groups [14, 16 − 18], and as it is clearly visible in figures 3 and 4, at shorter distances the Montecarlo data show a drastic deviation from the Nambu-Goto predictions. It would be interesting to understand if this signals the breakdown of the string picture itself, or if it is possible to describe these deviations in a string framework. In this respect, one would like to understand whether also the *deviations* from the Nambu-Goto behaviour follow a universal pattern. Preliminary results indicate that this is indeed the case for the Ising model and the SU(2) models in (2+1) dimensions, see [15]. A similar, but not completely coincident behaviour has been observed for the SU(3) model [15, 16]. Further numerical studies, auspiciously in a wider range of models, are needed to completely clarify this point. Nevertheless, we can ask ourselves which stringy mechanism could possibly be responsible for these deviations. A rôle could be played by contributions from higher genus surfaces bounded by the Polyakov loops; our formalism could help to test this possibility.

A second interesting possibility is that at short distance one is not any more allowed to neglect the Liouville mode in the analysis, and should resort to the full Polyakov formulation. To this purpose, the preliminary numerical results of [39] seem to suggest that the modification for shorter scales consists basically in a shift of the spectrum, in a way compatible with the effect of an extra degree of freedom.

JHEP07(2005)038

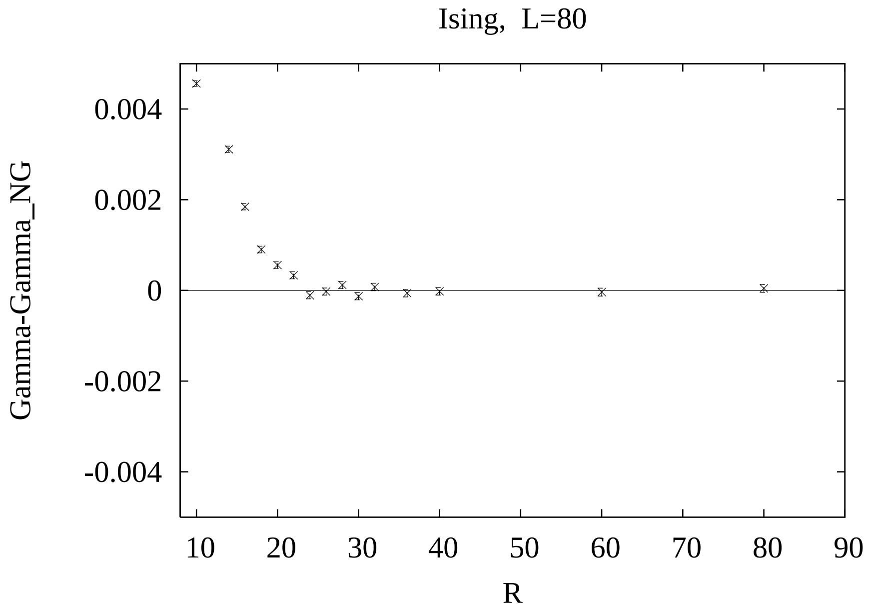

Figure 3: Montecarlo results for the Polyakov loop correlators in the (2+1) dimensional gauge Ising model. The data are taken at a fixed value of the lattice in the time direction: $L = 80$ (which corresponds to a very low temperature $T = T_c/10$) and a varying size of the interquark distance ($10 < R < 80$). In the figure is plotted the deviation of Γ (the ratio $G(R+1)/G(R)$ of two correlators shifted by one lattice spacing, see [16] for details) with respect to the Nambu-Goto string expectation Γ_{NG} (which with this definition of observables corresponds to the straight line at zero). Notice the remarkable agreement in the range $24 < R < 80$, which is not the result of a fitting procedure: in the comparison reported in the figure there is no free parameter (data taken from ref. [14] and [16]).

At first sight, including another degree of freedom seems to contradict the expected behaviour of the flux tube, whose relevant d.o.f. should be the $d-2$ transverse oscillations only. For this reason, after the discussions of the eighties and early nineties, no standard consistent quantization of the bosonic string seemed suited to describe the flux tube dynamics at all scales R: in $d < 26$, all exhibit unwanted features,[10] including the Polyakov string, which has an extra dimension parametrized by the Liouville field. Some interesting alternative proposals were made, such as, for instance, the effective string theory of Polchinski and Strominger [40], but a clear picture and a single candidate did not emerge.

More recently, in particular through the work of Polyakov [41] and the progresses made in supersymmetric Yang-Mills theory through the AdS/CFT correspondence [42], a different conceptual picture has been developed. The extra dimension parametrized by the Liouville field should represent the renormalization scale of the quantum gauge theory, rather than another space-time direction, thus evading the objection mentioned above to the relevance of the Polyakov model for the QCD string.

[10] In a non-covariant quantization of the free bosonic string Lorentz symmetry is broken, while in covariant quantization extra longitudinal modes appear.

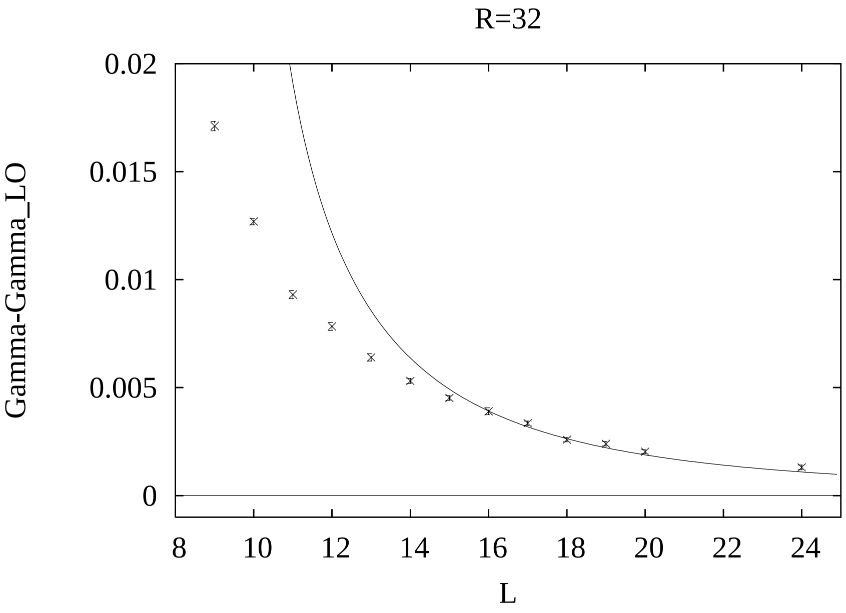

Figure 4: Montecarlo results for the Polyakov loop correlators in the (2+1) dimensional gauge Ising model. The data are taken at a fixed value $R = 32$ of the interquark distance and a varying size ($8 < L < 24$) of the lattice in the time directions. In the figure is plotted the deviation of Γ (defined as in the previous figure) with respect to the asymptotic free string expectation Γ_{LO} (which with this definition of observables corresponds to the straight line at zero). The curve is the Nambu-Goto prediction for this observable. Notice the remarkable agreement in the range $16 < L < 24$, which as for the previous figure is not the result of a fitting procedure: in the comparison reported in the figure there is no free parameter (taken from [16, figure 3]).

This paper aims not to tackle the difficult analysis of the relation between the full-fledged Polyakov formulation and gauge theories. Yet, having retrieved the large-R behaviour described by the Nambu-Goto effective model (and favoured by the numerical simulations) in a first-order formulation *à la* Polyakov, but neglecting the Liouville field, might prove useful in discussing the modifications due to its inclusion.

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